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Debt-equity decision-making with and without growth

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Abstract

Purpose – The purpose of this paper is to instruct upper level business students on the intricacies of the debt-equity choice with the emphasis on showing the interrelation of this choice with the plowback-payout choice.

Design/methodology/approach – The paper is designed around a pedagogical exercise that applies academic theories on the computation of the gain to leverage for an unleveraged nongrowth firm. A question and answer methodology is used within the exercise. The approach is instructional as it attempts to teach students about firm valuation and the variables that are important in the valuation process. The firm valuation method is based on perpetuity equations with and without growth.

Findings – Unlike an empirical study that concentrates on providing findings from a data analysis, this paper attempts to instill knowledge and skills to students when making debt-equity and plowback-payout choices.

Research limitations/implications – All gain to leverage equations used in this paper are limited by their derivational assumptions and the estimation of values for variables used in the equations.

Practical implications – Besides using the traditional Modigliani and Miller (MM)-Miller gain to leverage equations, this paper also uses more recent gain to leverage equations that attempt to bridge the gap between theory and practice by applying new theory on the impact of the plowback-payout choice on the debt-equity choice. Students will be able to compare traditional and recent gain to leverage equations and form their own opinions as to their potential value in practice. In the process, they should get an idea of the practical complexities of financial decision-making.

Social implications – Optimizing firm value through proper decision-making implies there is a proper and efficient utilization of societal resources.

Originality/value – The paper builds on a prior pedagogical paper that incorporated discount rates (costs of borrowing) within the nongrowth MM-Miller gain to leverage framework. This paper's originality and value lies in being the first pedagogical paper to incorporate growth as determined by the plowback-payout decision within the nongrowth gain to leverage framework.

Keywords Debt financing, Capital structure, Business education, Corporate finances

Paper type Technical paper

1. Introduction

Like capital budgeting, working capital management, and dividend policy, capital structure is a major topic taught in financial management courses. The capital structure question is: "How much debt (if any) is needed to optimize firm value?" The instructional exercise by Hull (2008) addresses this question by comparing gain to leverage (G_L) results using three perpetuity G_L equations. These equations come from the G_L models supplied by:

(1) Modigliani and Miller (1963), referred to as MM;

(2) Miller (1977); and

(3) the capital structure model (CSM) of Hull (2007).

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Through use of the CSM, the Hull (2008) exercise illustrates how the incorporation of costs of borrowing in a G_L equation causes G_L (and thus firm value) to increase as debt increases until G_L peaks at an optimal debt level before it begins falling as more debt is issued.

The three G_L equations used in the Hull (2008) exercise were all for a nongrowth situation where the firm's operating cash flows are fixed. A recent theoretical extension of the CSM by Hull (2010) has incorporated growth, thus making possible an instructional exercise of capital structure decision-making that concomitantly considers growth as rendered through the plowback-payout choice. In this paper, we use the CSM with growth to extend the nongrowth exercise of Hull (2008) by allowing a firm's operating cash flows to grow at a constant rate. For those familiar with the Hull (2008) exercise, we maintain continuity with that exercise by including the three G_L equations used previously. However, this paper's main focus revolves around the debt-equity choice for a growth firm.

The remainder of the paper is organized as follows. Section 2 looks at the motivation for this paper and describes learning objectives and means of assessment. Section 3 reviews CSMs focusing on the four perpetuity G_L equations used in this paper. Section 4 contains our instructional exercise of the capital structure decision-making process. Section 5 provides final remarks including supplementary teaching considerations. The appendices provide solutions to the questions contained in the capital structure decision-making exercise. Detailed excel solutions are available on request.

2. Background and assessment

2.1 Motivation for this exercise

This paper is motivated to incorporate growth within the nongrowth framework of the Hull (2008) instructional exercise. Further motivation comes from prior research (Leland, 1998; Graham and Harvey, 2001) that suggests capital structure decision-making cannot properly be taught because it lacks an adequate equation compared to well-accepted equations used to teach capital budgeting and the cost of capital. This research suggests that the models for capital budgeting (NPV, IRR, Payback, and PI) and costs of capital (WACC, dividend valuation model (DVM), and CAPM) are considered more reliable than G_L equations. In an attempt to overcome this "equation" problem, CSM equations have been developed by Hull (2007, 2010) to extend the MM (1963) and Miller (1977) G_L equations. While the MM-Miller research speak to the relation between debt and costs of borrowing, their G_L equations do not address how changes in these rates influence firm value with their G_L equations. The CSM research claims to fill in this void.

2.2 Incorporating growth into the nongrowth framework

The prior perpetuity G_L research (MM, 1963; Miller, 1977; Hull, 2007) provides no in depth analysis of the role of growth. To overcome this problem, Hull (2010) broadens the CSM framework by incorporating the plowback-payout choice with the debt-equity choice. In the process, he develops a number of new concepts including equilibrating unlevered and levered growth rates. Hull uses these two growth rates to get growth-adjusted discount rates that are needed to derive his G_L equation with growth. The Hull (2007, 2010) equations are brief, straightforward derivations from definitions of firm value. Like the original MM (1958, 1963) equations, they should not be judged

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so much on their capacity to garner a universal acceptance but on their potential to advance the debate and understanding of the impact of debt on firm value.

Growth-adjusted discount rates (used in the CSM) work in a fashion similar to the DVM with growth where a perpetual cash flow is divided by a discount rate minus a growth rate. However, unlike the DVM where the growth rate is not differentiated for unlevered and levered equity, the CSM with growth is only derived after growth rates for both unlevered and levered equity are first developed. The theoretical development of the unlevered and levered equity growth rates by Hull (2010) enables this instructional paper to extend the Hull (2008) pedagogical application, so that, educators are now offered a method of teaching capital structure decision-making that will be applicable to firms with growth.

2.3 Learning outcomes and assessment

By experiencing the exercise given in this paper, upper level business students with a sound background in corporate finance concepts should begin to understand the more advanced intricacies of capital structure decision-making. The following specific learning outcomes should result. First, students should learn how to compute four perpetuity G_L equations and compare these equations based on the different variables that each equation includes in its computation. These variables include tax rates, costs of borrowing, and growth rates. Second, for a growth firm, students should learn how the plowback-payout ratio choice affects the optimal debt-to-equity choice and firm valuation.

The above two outcomes fall within a school or program learning goal, such as “quantitative and scientific reasoning” and more general learning outcome for financial management courses, such as “computing present values” or “learning how to make (some specific) financial decision.” The outcome assessment process should include the identification of measures to assess learning, the analysis of the information given by these measures, and acting on this analysis to maintain and/or improve student performance. Thus, instructors seek to answer questions such as “How will students learn the desired outcome?” “How can we be sure they have learned it at some minimal level?” and “What can we do if some (or many) students have not achieved the minimal level?”

This paper’s pedagogical questions and solutions provide a means for instructors to assess whether students have mastered and achieved the stated (both general and specific) outcomes described above. This paper’s exercise has been modified over time through student feedback, so that, improvements have been made in the exercise to insure outcomes are attained by students at a satisfactory level.

3. Gain to leverage research

3.1 General research

Capital structure research is abundant and multifaceted (MM, 1958; Harris and Raviv, 1991; Myers, 2001; Mahrt-Smith, 2005; Hennessy and Whited, 2005; Strebulaev, 2007; Berk *et al.*, 2010; Matsa, 2010; Korteweg, 2010). This paper focuses on one aspect of this research: perpetuity G_L equations originating in the MM (1963) CSM.

3.2 MM and Miller equations for G_L

Assuming a nongrowth situation, MM assert that the gain to leverage (G_L) is:

$$G_L = T_C D \tag{1}$$

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where T_C is the effective corporate tax rate, $D = I/r_D$, I is the perpetual cash flow paid to debt owners, and r_D is the cost of debt. For MM, r_D is the risk-free rate (r_F). Equation (1) disregards personal taxes and leverage-related effects including those associated with financial distress.

Miller (1977) broadens equation (1) by examining the impact of debt from an investor's view after the payment of personal taxes on equity and debt income. The Miller equation is:

$$G_L = [1 - \alpha]D \tag{2}$$

where:

$$\alpha = \frac{(1 - T_E)(1 - T_C)}{(1 - T_D)}$$

T_E and T_D are the effective personal tax rates paid, respectively, by equity and debt owners, and now $D = (1 - T_D)I/r_D$ with r_D determined endogenously and $r_D > r_F$. At the firm level, G_L is zero for Miller when equation (2) is used. This is because the influence from personal taxes offsets the positive corporate tax shield effect while bankruptcy costs are considered inconsequential.

Empirical researchers (such as Warner, 1977; Altman, 1984; Kayhan and Titman, 2007) provide no perfect consensus concerning Miller's claim that leverage-related effects are insubstantial. Some researchers offer specific numbers concerning the positive effect of debt. For example, Graham (2000) estimates that the corporate and personal tax benefits of debt can increase firm value by as little as 4.3 percent with a mean incremental net benefit of 7.5 percent, while Korteweg (2010) finds that the net benefit of leverage averages 5.5 percent of firm value.

Theoretically, post-MM researchers favor optimal (or trade-off) CSMs. Beginning with Baxter (1967), Kraus and Litzenberger (1973), and Jensen and Meckling (1976), earlier optimal theorists argued that G_L is maximized only when a further issuance of debt does not cause the incremental wealth benefits of debt to be greater than its incremental costs. More recent theorists (Hennessy and Whited, 2005; Leary and Roberts, 2005; Korteweg, 2010) continue to advance this notion. However, direct and indirect costs from bankruptcy and agency effects (as discussed by optimal advocates) are numerous and arguably impossible to identify and measure with precision for all possible leverage-related effects.

The CSM research by Hull (2007, 2010) attempts to circumvent the impractical task of measuring the numerous agency-bankruptcy effects advanced by theoreticians. It does this by developing equations that require managers to only estimate tax, borrowing, and (if applicable) growth rates. However, a limitation of the CSM is that one must estimate these rates for each chosen debt level. Thus, any teaching exercise using the CSM must inform students that, while the exercise can provide illustrative worth, computing firm value at various leverage ratios in the real world is an estimation process that requires skill and experience.

3.3 CSM nongrowth and growth G_L Equations

Like MM and Miller, the CSM research focuses on an unlevered firm issuing perpetual debt to retire equity. For the nongrowth CSM situation with a plowback ratio (PBR) of zero, Hull (2007) shows:

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$$G_L = \left[1 - \frac{\alpha r_D}{r_L}\right] D - \left[1 - \frac{r_U}{r_L}\right] E_U \tag{3}$$

where:

α, r_D, and D	are as defined previously when describing equation (2).
r_U	is the exogenous cost of unlevered equity with $r_U > r_D$.
r_L	is the endogenous cost of levered equity with $r_L > r_U$.
V_U (or E_U)	is the unlevered equity value for a nongrowth firm referred to as V_U (nongrowth).

$$V_U(\text{nongrowth}) \text{ equals } \frac{(1 - T_E)(1 - T_C)C}{r_U} \text{ with } C = (1 - PBR)(CF_{BT})$$

where PBR is the before-tax plowback ratio (with PBR = 0 for the nongrowth situation) and CF_{BT} is the uncertain perpetual before-tax cash flow generated by operating assets. Thus, for the nongrowth situation, $C = CF_{BT}$.

A positive G_L as given by equation (3) can result strictly from designing security types, that are collectively more valued by investors as reflected in a lower overall cost of borrowing with some of the value explained by the tax advantage of debt. Hull (2010) argues that this positive valuation effect can be representable by an additional perpetuity cash flow that he refers to as "G." He describes G as enigmatic because G's exact perpetual cash flow depends on what rate it is discounted. He offers a way of computing G arguing that its calculation is important due to its influences on the growth rate of levered equity (g_L). Using g_L as his key concept, Hull (2010) extends equation (3), so that, G_L with growth is:

$$G_L = \left[1 - \frac{\alpha r_D}{r_{L_g}}\right] D - \left[1 - \frac{r_{U_g}}{r_{L_g}}\right] E_U \tag{4}$$

where:

α, r_D, and D	are as defined previously.
r_{Ug}	is the growth-adjusted discount rate on unlevered equity given as $r_{Ug} = r_U - g_U$ with r_U as the unlevered cost of equity and g_U as the unlevered equity growth rate.
r_{Lg}	is the growth-adjusted discount rate on levered equity given as $r_{Lg} = r_L - g_L$ with r_L as the levered cost of equity and g_L as the levered equity growth rate.
V_U (or E_U)	is the unlevered equity value for a growth firm referred to as V_U (growth).

$$V_U(\text{growth}) \text{ equals } \frac{(1 - T_E)(1 - T_C)C}{r_{U_g}}$$

where $C = (1 - PBR)(CF_{BT})$ with $C < CF_{BT}$ because $PBR > 0$ for a growing firm using internal equity (or retained earnings). V_U (growth) in equation (4) differs from V_U

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(nongrowth) in equation (3) because PBR > 0 in equation (4) while PBR = 0 in equation (3). Because PBR + POR = 1, we can also express C as C = POR(CF_{BT}) where POR is the payout ratio. For Hull (2010), PBR and POR are defined on a before-tax basis.

Hull (2005) offers a proof for an equation similar to equation (4) but, unlike Hull (2010), does not incorporate PBR in his formulation and also does not define g_L in terms of G. In Table I of our instructional exercise, formulas for computing g_U and g_L will be given. As discussed by Hull (2010), equation (4) is the most general equation as it reduces to equation (3) when growth rates are zero, just as equation (3) reduces to equation (2) if differences in costs of capital are ignored and to equation (1) if personal tax rates are also ignored.

4. Instructional exercise

Like the Hull (2008) instructional exercise, this paper uses the three nongrowth G_L equations: MM (1963), Miller (1977), and Hull (2007). The distinguishing feature of this paper's exercise is the use of the G_L equation with growth given by Hull (2010). Incorporating growth into capital structure decision-making exercise unfortunately involves some complexity. Consequently, familiarity with the CSM growth research is helpful even though not absolutely necessary to conduct the exercise (just as a full understanding of the Black and Scholes model is helpful but not necessarily essential to instruct students on its general computation and interpretation).

Instructors will find six sets of questions in this paper's exercise. At their discretion, instructors may omit questions either to condense the exercise or to tailor it to fit the degree of difficulty desired. Besides the answers provided in Appendices 1-6, Excel spreadsheets with more detailed solutions are available on request. For the convenience of those familiar with the nongrowth pedagogical exercise of Hull (2008), we use (where applicable) the same values for variables used in that exercise and the first two questions of this paper's exercise cover the essence of that exercise. An instructor will also notice that costs of borrowings are given in this paper's exercise whereas, Hull (2008) had students compute these values using the CAPM. This is done for simplicity and to focus on the growth issues. Instructors who do not like this omission are referred to the Hull (2008) exercise where equity and debt betas are given from which students can compute costs of borrowing. Hull (2010) uses costs of borrowings influenced by Pratt and Grabowski (2008), and so, values for his costs of borrowing can differ from this paper's values. He also uses slightly different tax rates and assumes a 4-percent riskless rate, while this paper keeps the 5 percent used by Hull (2008).

4.1 Question 1: computing MM and miller values

Unlevgrowth, Inc. (UGI) is an unlevered growth firm. UGI's managers believe it can increase its equity value by retiring a proportion of its outstanding equity through a new debt issue. UGI will treat its new debt as perpetual since it plans to continuously roll it over whenever it reaches maturity. For its first task, UGI's managers want to compute the valuation impact using the G_L equations supplied by MM and Miller. To complete this task, UGI's managers estimate values for variables when using the MM and Miller G_L equations. These values along with relevant formulas are included in Table II:

(a) Answer the below questions using the MM viewpoint and assumptions:

(i) From the information in Table II, what is MM's unlevered equity value ($V_{U_{MM}}$)?

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Table I.
CSM values

PBR = plowback ratio used on CF_{BT} = 0.35	POR = payout ratio = 1 − 0.35 = 0.65
T_C = corporate tax rate = 30.00 percent	T_D = personal tax rate on debt income = 15.00 percent
α = 0.7823529411765	r_U = 11.00 percent

CF_{BT} = perpetual before-tax cash flow generated by operating assets = \$1,654,135,338.34
RE = before-tax retained earnings = PBR(CF_{BT}) = 0.35(\$1,654,135,338.34) = \$578,947,368.42
C = before-tax cash to equity = (1 − PBR)(CF_{BT}) = 0.65(\$1,654,135,338.34) = \$1,075,187,969.92
I = Interest = $r_D D/((1 - T_D))$ where D is D_{Miller} (computed for each D choice)
Note: Because D considers personal taxes and is thus D_{Miller}, D must be divided by $(1 - T_D)$ because the below g_L equation uses an "I" value before personal taxes are considered as this is the same "I" that lowers a firm's taxable income
G = perpetuity cash flow (besides I) created with debt when $G_L \neq 0$ (supplied for each D choice)
g_U = unlevered equity growth rate = $(r_U(1 - T_C)RE)/C = (0.11(1 - 0.3)$ $\$529,323,308)/\$1,124,812,030 = 4.146153846$percent
g_L = levered equity growth rate = $(r_L(1 - T_C)RE)/C + G - (I/(1 - T_C))$ (computed for each D choice)
r_{Ug} = growth−adjusted unlevered equity rate of return = $r_U - g_U$ = 11percent − 4.146154percent = 6.853846154percent
r_{Lg} = growth-adjusted levered equity rate of return = $r_L - g_L$ (computed for each D choice)
V_U (nongrowth) = $((1 - T_E)(1 - T_C)(1 - PBR)CF_{BT})/r_U = ((1 - 0.05)(1 - 0.3)(1 - 0)$ $\$1,654,135,338)/0.11 = \$10,000,000,000$.
V_U(growth) = $((1 - T_E)(1 - T_C)(1 - PBR)CF_{BT})/r_{Ug} = ((1 - 0.05)(1 - 0.3)(1 - 0.35)$ $\$1,654,135,338)/0.06853846154 = \$10,432,098,765.43$.

Notes: Some values from Table II are repeated; other values are changed by growth; and, some are now supplied

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Table II.
MM and Miller values

$T_{E_{MM}}$ = personal tax rate on equity income = 0 percent	$T_{E_{Miller}}$ = 5.00 percent
$T_{D_{MM}}$ = personal tax rate on debt income = 0 percent	$T_{D_{Miller}}$ = 15.00 percent
T_C = corporate tax rate = 30.00 percent	$\alpha_{Miller} = (1 - T_{E_{Miller}})(1 - T_C)/(1 - T_{D_{Miller}})$
r_U = cost of capital for unlevered equity = 11.00 percent	r_F = risk-free rate = 5.00 percent
PBR = plowback ratio used on CF_{BT} (PBR = 0 with nongrowth)	POR = payout ratio = 1 − PBR
CF_{BT} = perpetual before-tax cash flow generated by operating assets = \$1,654,135,338.34	
RE = before-tax retained earnings = PBR(CF_{BT}) with RE = \$0 for nongrowth when PBR = 0	
C = before-tax cash to equity = (1 − PBR)(CF_{BT}) with C = CF_{BT} for nongrowth when PBR = 0	
I = Interest = r_D(D) where I = 0 for an unlevered firm because D = 0	
$V_{U_{MM}} = ((1 - T_{E_{MM}})(1 - T_{C_{MM}})C)/r_U$ where C = (1 − PBR)(CF_{BT})	$V_{U_{Miller}} = (1 - T_{E_{Miller}})V_{U_{MM}}$
$D_{MM} = P(V_{U_{MM}})$ where P = proportion of $V_{U_{MM}}$ retired by D_{MM}	$D_{Miller} = (1 - T_{E_{Miller}})D_{MM}$
$G_{L_{MM}} = T_C(D_{MM})$	$G_{L_{Miller}} = [1 - \alpha_{Miller}]D_{Miller}$

Note: When different, the MM and Miller values are denoted in subscripts

(ii) What is the dollar amount of the MM debt (D_{MM}) that will be issued if UGI retires 0.5 of its unleveraged firm value (e.g. retires 0.5 of $V_{U_{MM}}$)?

(iii) What is the MM gain to leverage ($G_{L_{MM}}$) from retiring 0.5 of $V_{U_{MM}}$?

(iv) What is the MM debt-to-firm value ratio ($D_{MM}/V_{L_{MM}}$) after retiring 0.5 of $V_{U_{MM}}$ (where $V_{L_{MM}} = V_{U_{MM}} + G_{L_{MM}}$)?

(b) Answer the first four questions from (i) through (iv) in part (a) except now use the Miller values that consider an investor's viewpoint after they pay personal taxes? In other words, what are $V_{U_{Miller}}$, D_{Miller}, $G_{L_{Miller}}$, and $D_{Miller}/V_{L_{Miller}}$?

(c) UGI's managers decide it is better to look at other debt choices besides just $P = 0.5$. In particular, UGI wants MM and Miller G_L values for nine choices that retire from 0.1 to 0.9 of its unlevered equity with increasing increments of 0.1 as shown in Table III. Using Excel (or a similar software to expedite the computational process), repeat parts (a) and (b) for all debt choices (e.g. for all P-values) given in Table III, so, you can fill in all of Table III's blank cells. Is there an optimal leverage ratio for MM and Miller? Explain.

4.2 Question 2: computing CSM values without growth

UGI is not satisfied with the results from MM and Miller models because it believes its predicted debt choice is unrealistic. Thus, UGI's managers decide to turn to the CSM without growth. This CSM nongrowth equation is:

$$G_L(\text{nongrowth}) = \left[1 - \frac{\alpha r_D}{r_L}\right]D - \left[1 - \frac{r_U}{r_L}\right]V_U.$$

To use this equation, UGI estimates the costs of capital (r_D and r_L) for each debt choice. The values for r_D and r_L are given in Table IV. The CSM nongrowth value for V_U and

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Table III. MM and Miller values for debt choices

Variables	P = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$D_{MM} = P(V_{U_{MM}})$	1.0526								9.4737
$G_{L_{MM}}$	0.3158								2.8421
$D_{MM}/V_{L_{MM}}$	0.0971								0.7087
$D_{Miller} = P(V_{U_{Miller}})$	1.0000								9.0000
$G_{L_{Miller}}$	0.2176								1.9588
$D_{MM}/V_{L_{Miller}}$	0.0979								0.7526

Notes: While answers placed in Table III are rounded off, it is best to not round-off until after you have performed all computations; otherwise, errors can occur; values for the "0.1" and "0.9" columns have been put in Table III to help jump-start the process; also, you can put in your answers from parts (a) and (b) for the "0.5" column; where applicable, values are expressed in billions of dollars and to four decimal places

Table IV. CSM values for debt choice with nongrowth

Variables	P = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
V_U (or E_U)	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
$D = P(V_U)$	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000
r_D (%)	5.06	5.30	5.60	6.02	6.62	7.34	8.18	9.14	10.28
r_L (%)	11.12	11.36	11.84	12.50	13.28	14.30	15.50	16.88	18.44
G_L	0.5361								1.0400
V_L	10.5361								11.0400
D/V_L	0.0949								0.8152

Notes: As before, it is best not to round-off numbers until after you have performed all computations; otherwise, errors can occur; values for the "0.1" and "0.9" columns are given to jump-start the computational process; to compute values for any column, first compute the CSM nongrowth G_L equation and then compute V_L given by $V_L = V_U + G_L$; where applicable, values are expressed in billions of dollars and to four decimal places

D in Table IV are the same as Miller's V_U and D values because the CSM (like Miller) also considers both personal and corporate taxes. Answer the below questions:

- Fill in the blank cells in Table IV.
- Identify and comment on the debt choice (or P-value) for UGI's maximum G_L, maximum V_L, and the optimal D/V_L.

4.3 Question 3: computing growth-adjusted costs of borrowing

UGI believes if it can improve its unlevered firm value through a new line of marketable products for which future patents can assure constant long-term growth in cash payable to equity. UGI estimates that a long-term before-tax PBR of 0.35 will sustain its growth objectives. If growth adds to firm value, UGI will then use the G_L equation given by the CSM with growth to determine if leverage can further enhance its value beyond that computed for its nongrowth leveraged situation. To use the CSM equation with growth, UGI must first estimate the levered growth rates (g_L) for its desired debt choices. Using the values and equations in Table I, supply answers to the below questions:

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- From the V_U (nongrowth) and V_U (growth) values computed in Table I, one can see that UGI increases its value through growth from undertaking its new line of products? Explain how this occurs?
- Fill in all blank cells in Table V using the values and equations supplied in Table I and Table V.
- What does a negative g_L value suggest?

4.4 Question 4: computing CSM values using the CSM with growth

Having estimated g_L values for each debt choice, UGI is now ready to determine its optimal debt choice using the G_L equation for the CSM with growth. This equation is:

$$G_L(\text{growth}) = \left(1 - \frac{\alpha r_D}{r_{L_g}}\right)D - \left[1 - \frac{r_{U_g}}{r_{L_g}}\right]E_U.$$

Answer the below questions:

- Fill in the blank cells in Table VI. For the r_{Lg} row, copy in the values that you previously computed. Identify the debt choice (or *P*-value) for UGI's maximum G_L, maximum V_L, and the optimal D/V_L.
- Compare the optimal debt choice and maximizing values in part (a) for Table VI with those in Table IV (where the CSM nongrowth G_L equation was used). Was growth a good choice? Explain.

4.5 Question 5: computing and comparing G_L values

UGI wants to compare its G_L values computed from questions 1, 2, and 4 where it used the MM, Miller, and two CSM equations for the nine debt choices ($P = 0.1$ through $P = 0.9$). Answer the below questions:

	P = Debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
Variables	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
V_U (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
$D = P(V_U)$	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
G	0.0544	0.1022	0.1407	0.1773	0.2188	−0.9366	−0.7763	−0.6134	−0.4654
r_D (%)	5.06	5.30	5.60	6.02	6.62	7.34	8.18	9.14	10.28
r_L (%)	11.12	11.36	11.84	12.50	13.28	14.30	15.50	16.88	18.44
I	0.0621								1.1355
g_L (%)	4.330								−7.382
r_{Lg} (%)	6.790								25.823

Notes: To avoid rounding off errors, use the more exact values for V_U, D, g_L, r_{Lg}, and G; for the latter, use the following G values that correspond to respective debt choice (or *P*-values) from 0.1 to 0.9: \$54,381,590, \$102,153,829, \$140,719,080, \$177,341,522, \$218,817,110, −\$936,605,610, −\$776,316,593, −\$613,473,171, and −\$465,392,463; values for the "0.1" and "0.9" columns are given to jump-start the computational process; the CSM with growth offers the concept of "G" so as to express the gain to leverage (G_L value) as a perpetuity; thus, a negative or positive sign for G corresponds to the sign of G_L; this will be seen in the next problem when we compute G_L values; where applicable, values are expressed in billions of dollars and to four decimal places

Table V.
CSM values for debt choices with growth

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Variables	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	P = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
V_U (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
$D = P(V_U)$	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
r_D (%)	5.06	5.30	5.60	6.02	6.62	7.34	8.18	9.14	10.28
r_{Lg} (%)	6.790								25.82
G_L	0.5326								−1.1985
V_L	10.9647								9.2336
D/V_L	0.0951								1.0168

Notes: As before, it is best not to round-off numbers until you are ready to put them in the table; values for the "0.1" and "0.9" columns are given to jump-start the computational process; to compute values for any column, begin by computing G_L using the CSM equation with growth; next, compute V_L given by $V_L = V_U + G_L$; where applicable, values are expressed in billions of dollars and to four decimal places

Table VI. CSM values for debt choices with growth

- From your G_L answers in Tables III, IV and VI, fill in Table VII. In comparing the four G_L values for each debt choice from all four G_L equations, which equations are consistent with trade-off (or optimal) theory? Explain.
- Examine the G_L values for the first four debt choices ($P = 0.1$ through $P = 0.4$) and compare them in terms of dollar amounts. Based on your comparison of G_L values, would you conclude that a positive tax shield effect is the only explanation for a positive G_L? Explain.
- Which equation would you feel more comfortable with if you were a UGI manager charged with the capital structure decision? Explain.

4.6 Question 6: comparing results for different PBR choices

While UGI's managers have chosen a PBR of 0.35, they are curious how other PBRs influence UGI's growth rates, firm value, and debt choice. Thus, they repeat their previous computations using other PBRs with a sample of their results given in Table VIII. Answer the below questions:

- Table VIII does not provide results for a PBR less than 0.30. Illustrate why UGI's unlevered value (V_U) does not change with a PBR of 0.30 and how its V_U value falls if the PBR falls under 0.30. (Hint: all values needed are given in Tables I and II. Begin by computing $RE = PBR(CF_{BT})$ and $C = (1 - PBR)(CF_{BT})$ using your chosen PBR (say 0.25). You then compute g_U from the formula given in Table I,

G_L model	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	P = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
MM's G_L									
Miller's G_L									
CSM's G_L (nongrowth)									
CSM's G_L (growth)									

Note: Express your answers in billions of dollars and to four decimal places

Table VII. Comparison of values given by four G_L equations

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using your values for RE and C as well as values for T_E, T_C, and r_U of 5, 30, and 11.00 percent given in Table II. Given g_U, you can compute V_U (growth) using the formula in Table I. Repeat for PBR = 0.3. You will find that any PBR < 0.30 will give a V_U (growth) less than the \$10B value found both for V_U (nongrowth) with PBR = 0 and for V_U (growth) with a PBR = 0.30).

- From the values in Table VIII, is it possible to identify a PBR that maximizes firm value (V_L) for UGI? Does this PBR depend on UGI being able to sustain g_U? Explain.
- Do the PBRs and debt choices in Table VIII indicate there is one plowback-payout choice and one debt-equity choice that together maximize firm value? Explain.

5. Final remarks

Student feedbacks when doing exercises involving CSM equations have been positive over the years from both upper level undergraduate finance students and graduate students. The below quote is from a student who has experienced the exercise and is representative of students' comments from taking the exercise:

> The CSM with growth model is most complete because it provides for more scenarios, thus being consistent with diversity of situations faced by managers in trying to determine the optimum leverage. The CSM framework can give due consideration to the growth that can be brought about if the company keeps aside some of its earning to fuel expansion.

The approval from students concerning the CSM applications has been received not only for upper level corporate finance courses taught in the classroom but also a graduate level course taught online.

Before presenting the exercise, we have found it advantageous to incorporate the exercise's formulas within one's lectures and handouts for which ample time must be allotted. By doing this, students will know what to expect and see that the equations can be readily used to generate answers to computational questions. However, since the exercise involves nine debt choices with repeated computations, we suggest that instructors use this exercise to also enhance a student's Excel spreadsheet skills.

To encourage student interaction and lower the amount of work, teachers can conduct the teaching exercise by assigning students to teams. See Hull *et al.* (2007) who offer some particulars when conducting a team exercise involving the collaborative aspects of peer learning. More can be expected when student works in teams. Within teams,

PBR	g_U (%)	g_L (%)	V_U	V_L	Debt choice
0.30	3.30	7.59	10.0000	12.3442	0.60
0.34	3.97	7.15	10.3223	12.6419	0.50
0.35	4.15	7.54	10.4321	12.9677	0.50
0.36	4.33	7.95	10.5567	13.3616	0.50
0.37	4.52	8.38	10.6981	13.8445	0.50
0.38	4.72	7.10	10.8588	13.1821	0.40
0.50	7.70	7.70	16.6667	16.6667	0.00

Note: Where applicable, express values in billions of dollars and to four decimals

Table VIII.
Computing variable values for optimal PBRs and debt choices

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there is more likelihood that at least one or two students will have advanced skills in Excel and PowerPoint and thus can more likely produce visual aids with tables, charts, and graphs to illustrate the optimal leverage choice and how G_L changes with the plowback choice or debt choice. Hull (2010) supplies two examples of graphs that could be done in Excel showing the influence of the plowback and debt choices. Alternately, student teams could be commissioned to find a desirable plowback and debt choice for a case study of an individual firm. A case study would challenge students to apply a CSM equation to a real firm of their choice (or a firm assigned by the instructor). Hull (2005) offers a procedure to unlever a firm so that a CSM equation can be used.

To accompany this paper's exercise, we have spreadsheets (identified by their tab names) that can be sent electronically by requesting them. These spreadsheets not only give detailed solutions to the questions asked in the previous section, but also solutions to simulatory applications of equation (4) when a variable is changed. Values for variables in this paper's exercise can be easily modified when using the provided Excel spreadsheets if instructors want to choose values for variables based on their own beliefs. For example, one can modify one cell for the corporate tax rate or PBR in a spreadsheet and see how (at the stroke of a single key) a change in this variable's value affects the maximum G_L and the optimal debt choice. The spreadsheets strive to give sufficient details so that instructors and students will hopefully find them easy to use and understand. In particular, we highlight the solutions to the assigned questions and provide explanatory notes so the user can follow the computations that are being performed. We can note that any of these spreadsheets can be easily adapted by using costs of capital other than those generated by the CAPM. For example, Hull (2010) uses costs of capital influenced by the research of Hull (2005, 2007) and betas and debt ratings given by Pratt and Grabowski (2008). The make-up of these spreadsheets is similar to those discussed by Hull (2008) except they focus on equation (4) instead of equation (3). The CSM applications found in these spreadsheets point out the potential influence of external factors (such as monetary and legislative policies) on the debt decision as well as market factors (such as signaling and agency considerations). For brevity's sake, the findings of these supplementary applications are not reported in this paper but are available on request.

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Further reading

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Appendix 1. Solutions to question 1

(a) The below answers use the MM (1963) model that looks at value from a firm's viewpoint ignoring personal taxes:

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(i) Noting that $C = (1 - PBR)(CF_{BT}) = (1 - 0)(\$1{,}654{,}135{,}338.34) = \$1{,}654{,}135{,}338.34$ and inserting this value and our other given values from Table II into:

$$V_{U_{MM}} = \frac{(1 - T_{E_{MM}})(1 - T_{C_{MM}})C}{r_U}$$

we have:

$$V_{U_{MM}} = \frac{(1 - 0)(1 - 0.3)(\$1,654,135,338.34)}{0.11} = \frac{0.7(\$1,654,135,338.34)}{0.11}$$
$$= \frac{\$1,157,894,736.84}{0.11} = \$10,526,315,789.$$

(ii) Because, UGI retires 0.5 of its unlevered equity value, its debt choice represented as P (the proportion of $V_{U_{MM}}$ retired) is 0.5. Expressing MM's debt as D_{MM}, we get:

$$D_{MM} = P(V_{U_{MM}}) = 0.5(\$10,526,315,789) = \$5,263,157,895.$$

(iii) For MM, we use $G_L = T_C D$. Referring to G_L as $G_{L_{MM}}$ (the MM before-personal tax gain from leverage) and D as D_{MM} (the MM before-personal tax value of debt), we have $G_{L_{MM}} = T_C D_{MM}$. Inserting in the Table II value of $T_C = 0.30$ and the value of $D_{MM} = \$5{,}263{,}157{,}895$ just computed in the previous problem, we have:

$$G_{L_{MM}} = T_C D_{MM} = 0.3(\$5,263,157,895) = \$1,578,947,368.$$

(iv) Noting that MM's leveraged firm value $= V_{L_{MM}} = V_{U_{MM}} + G_{L_{MM}}$, we get MM's debt-to-firm value ratio:

$$\frac{D_{MM}}{V_{L_{MM}}} = \frac{D_{MM}}{V_{U_{MM}} + G_{L_{MM}}} = \frac{\$5,263,157,895}{\$10,526,315,789 + \$1,578,947,368} = 0.4348.$$

(b) The below answers use the Miller (1977) model that looks at value from an investor's viewpoint that considers personal taxes.

(i) Recognizing that $V_{U_{Miller}} = (1 - T_{E_{Miller}})V_{U_{MM}}$, we have: $V_{U_{Miller}} = (1 - 0.05)$ $\$10{,}526{,}315{,}789 = \$10{,}000{,}000{,}000$ or \$10 billion. We could also modify the previous MM equation for V_U with the Miller numbers to get the equation of:

$$V_{U_{Miller}} = \frac{(1 - T_{E_{Miller}})(1 - T_{C_{Miller}})(C)}{r_U}.$$

Inserting the values in Table II into this equation gives the same \$10 billion answer.

(ii) We have:

$$D_{Miller} = (1 - T_{E_{Miller}})D_{MM} = (1 - 0.05)\$5,263,157,895 = \$5,000,000,000.$$

We get the same answer using:

$$D_{Miller} = P(V_{U_{Miller}}) = 0.5(\$10,000,000,000) = \$5,000,000,000.$$

(iii) For Miller, we have: $G_L = [1 - \alpha]D$. Referring to G_L as $G_{L_{Miller}}$ (the Miller after-personal tax gain from leverage), α as α_{Miller} (the value for α using $T_{E_{Miller}}$ and $T_{D_{Miller}}$), and D as D_{Miller} (the Miller after-personal tax value of debt), we have: $G_{L_{Miller}} = [1 - \alpha_{Miller}]D_{Miller}$. Using:

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$$\alpha_{Miller} = \frac{(1 - T_{E_{Miller}})(1 - T_C)}{(1 - T_{D_{Miller}})} = \frac{(1 - 0.05)(1 - 0.30)}{(1 - 0.15)}$$

$$= 0.7823529411765 \text{ and } D_{Miller} = \$5,000,000,000$$

from part (ii), we have:

$$G_{L_{Miller}} = [1 - \alpha_{Miller}]D_{Miller} = [1 - 0.7823529411765]\$5,000,000,000$$
$$= \$1,088,235,294.$$

(iv) Noting Miller's leveraged firm value = $V_{L_{Miller}} = V_{U_{Miller}} + G_{L_{Miller}}$, we get:

$$\text{Miller's debt–to–firm value ratio} = \frac{D_{Miller}}{V_{L_{Miller}}} = \frac{D_{Miller}}{V_{U_{Miller}} + G_{L_{Miller}}}$$

$$= \frac{\$5,000,000,000}{\$10,000,000,000 + \$1,088,235,294}$$

$$= 0.4509.$$

(c) We begin by copying in our answers for the "0.5" column in Table AI that were previously computed in parts (a) and (b). We then follow the same computational procedure used in parts (a) and (b) to get the desired answers for the other columns of Table AI. As seen in Table AI, the MM and Miller equations both suggest that more debt is better. Thus, there is no optimal leverage ratio in the trade-off sense of G_L increasing with debt before decreasing. For a finite set of choices, the optimal leverage ratio is the one with the most debt, which would be the values in the last column for the "0.9" debt choice that retires 90 percent of the unlevered firm value. As seen in this column the debt-to-firm value ratios are 0.7087 for MM and 0.7526 for Miller. Both of these debt-to-firm value ratios greatly exceed the debt-to-firm value ratio for a typical firm, suggesting that the MM and Miller G_L equations fail to capture the negative leverage-related effects that govern the real world with its frictions that include bankruptcy and agency costs. In reality, most firms would only achieve such high leverage ratios unintentionally as when the value of their equity falls due to earnings problems. Furthermore, lenders would likely stop lending money to firms before such high leverage ratios could be reached (and even then lenders would charge an exorbitantly high interest rate that firms would not likely afford).

Table AI.
MM and Miller values for debt choices

	P = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
Variables	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$D_{MM} = P(V_{U_{MM}})$	1.0526	2.1053	3.1579	4.2105	5.2632	6.3158	7.3684	8.4211	9.4737
$G_{L_{MM}}$	0.3158	0.6316	0.9474	1.2632	1.5789	1.8947	2.2105	2.5263	2.8421
$D_{MM}/V_{L_{MM}}$	0.0971	0.1887	0.2752	0.3571	0.4348	0.5085	0.5785	0.6452	0.7087
$D_{Miller} = P(V_{U_{Miller}})$	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000
$G_{L_{Miller}}$	0.2176	0.4353	0.6529	0.8706	1.0882	1.3059	1.5235	1.7412	1.9588
$D_{MM}/V_{L_{Miller}}$	0.0979	0.1917	0.2816	0.3680	0.4509	0.5307	0.6075	0.6814	0.7526

Note: Where applicable, values are expressed in billions of dollars and to four decimal places

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Appendix 2. Solutions to question 2

(a) We fill in all empty cells in Table IV.

(b) Table AII reveals that the maximum G_L is \$1.3331B. This G_L occurs in the "0.5" column, which is the column that represents a debt choice (or *P*-value) of 0.5. This column also contains the greatest V_L of \$11.3331B. Moving down the "0.5" column, we find that the optimal debt-to-firm value ratio (D/V_L) is 0.4412. The ratio of 0.4412 denotes that UGI should finance its projects with \$44.12 of debt for every \$100 of total financing from both debt and equity. Table AII shows that if UGI chooses more than \$5.0000B in debt, both the gain to leverage (G_L) and firm value (V_L) fall. Thus, UGI's value is maximized at issuing \$5 billion in debt. Subsequent values for the G_L and V_L rows become increasingly less positive indicating even greater leverage-related costs as the debt choice increases.

Below we illustrate the computations in Table AII for the "0.5" column, which is the column where UGI maximizes its value by issuing \$5.0000B in debt to retire one-half its equity. Using the CSM G_L equation and inserting the previous values (including $\alpha = 0.7823529411765$ from Table II), we have:

$$G_L(0.5\ \text{column}) = \left[1 - \frac{\alpha r_D}{r_L}\right]D - \left[1 - \frac{r_U}{r_L}\right]V_U$$

$$= \left[1 - \frac{0.7823529411765(0.0662)}{0.1328}\right]\$5B - \left[1 - \frac{0.11}{0.1328}\right]\$10B \longrightarrow$$

$$G_L(0.5\ \text{coloumn}) = \$3,050,008,859 - \$1,716,867,470 \rightarrow G_L = \$1,333,141,389 \text{ or about } \$1.3331B.$$

We can compute the value of the levered firm for the 0.5 debt choice (or *P*-value) as:

$$V_L(0.5\ \text{column}) = V_U + G_L(0.5\ \text{column}) = \$10,000,000,000 + \$1,333,141,389 \longrightarrow V_L = \$11,333,141,389 \text{ or about } \$11.3331B.$$

Finally, we compute the debt-to-firm value ratio for 0.5 debt choice. Doing this, we get:

$$\frac{D(0.5\ \text{column})}{V_L(0.5\ \text{column})} = \frac{\$5.000B}{\$11.3331B} = 0.4411839 \rightarrow \frac{D}{V_L} = \text{about } 0.4412.$$

Appendix 3. Solutions to question 3

(a) As seen in Table I, growth increases UGI's unlevered value from V_U (nongrowth) of \$10,000,000,000 to V_U (growth) of \$10,432,098,765.43. UGI increases its value by using its

Variables	*P* = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
V_U (or E_U)	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
$D = P(V_U)$	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000
r_D (%)	5.06	5.30	5.60	6.02	6.62	7.34	8.18	9.14	10.28
r_L (%)	11.12	11.36	11.84	12.50	13.28	14.30	15.50	16.88	18.44
G_L	0.5361	0.9531	1.1804	1.2929	*1.3331*	1.2829	1.2066	1.1276	1.0400
V_L	10.5361	10.9531	11.1804	11.2929	*11.3331*	11.2829	11.2066	11.1276	11.0400
D/V_L	0.0949	0.1826	0.2683	0.3542	*0.4412*	0.5318	0.6246	0.7189	0.8152

Note: Where applicable, values are expressed in billions of dollars and to four decimal places

Table AII. CSM values for debt choice with nongrowth

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before-tax PBR of 0.35 as given in Table I. One obvious and simple answer as to how a PBR of 0.35 increases value would be that the earnings plowed back are more valuable than what investors could achieve by having it paid out as cash. In other words, the value of the cash flows from retained earnings is greater than the value obtained from just paying it out.

(Note: a more complete answer is not obvious unless one understands the exact valuation impact that a PBR of 0.35 achieves with all expenses considered. In the developing the CSM with growth, Hull (2010) argues that the costs from retaining earnings for growth are determined by the double taxation at the effective corporate tax rate (T_C). The "first corporate taxes paid" occurs before retained earnings are invested (this cost does not hold with external equity). The "second corporate taxes paid" comes later when the cash flows created from the retained earnings are also taxed at the corporate level before paid out as dividends (this cost holds with external equity). To keep nongrowth firm value for an unlevered firm equal to its growth value, Hull (2010) shows that the minimum PBR needed so that firm value does not fall must be T_C. This implies that the minimum g_U = (minimum PBR)(r_U) = $T_C(r_U)$. This is shown as follows. With $T_C = 0.3$ and $r_U = 0.11$, we get: minimum PBR = T_C = 0.30 and minimum g_U = (minimum PBR)(r_U) = $T_C(r_U) = 0.3(0.11) = 0.0330$. Using the minimum PBR of 0.30 for PBR and the minimum g_U of 0.0330 for g_U in the V_U (growth) equation given in Table I and expressing r_{Ug} as $r_U - g_U$, we have:

$$V_U(\text{growth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U}$$

$$= \frac{(1-0.05)(1-0.3)(1-0.3)\$1,654,135,338}{0.11-0.0330} = \$10,000,000,000.$$

This value of \$10B is the same value found in Table I for V_U (nongrowth). Keep in mind, this equation assumes the use of internal equity. If the firm used external equity, Hull (2010) claims that a firm is able to avoid the double corporate taxation from the use of retained earnings such that the minimum g_U could be much less than 0.0330 because the issuance costs associated with external equity are, on average, only about one-fifth of the costs associated with paying an extra corporate tax. Hull argues that less costs using external equity explains the recent strong empirical evidence against the pecking order theory.)

(b) We fill in all empty cells in Table AIII. Below we illustrate the computations for I, g_L, and r_{Lg} when the debt choice is 0.5. All other computations for these three variables are computed in the same fashion for each debt choice (or *P*-value).

Precise value for $D = P(V_U) = 0.5(\$10,432,098,765.43) = \$5,216,049,382.72$ or about \$5.2160B.

	P = debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
Variables	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
V_U (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
$D = P(V_U)$	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
G	0.0544	0.1022	0.1407	0.1773	0.2188	−0.9366	−0.7763	−0.6134	−0.4654
r_D (%)	5.06	5.30	5.60	6.02	6.62	7.34	8.18	9.14	10.28
r_L (%)	11.12	11.36	11.84	12.50	13.28	14.30	15.50	16.88	18.44
I	0.0621	*0.1301*	*0.2062*	*0.2955*	*0.4062*	*0.5405*	*0.7028*	*0.8974*	1.1355
g_L (%)	4.330	*4.643*	*5.208*	*6.101*	*7.541*	*−9.147*	*−8.909*	*−8.340*	−7.382
r_{Lg} (%)	6.790	*6.717*	*6.632*	*6.399*	*5.739*	*23.447*	*24.409*	*25.230*	25.823

Note: Where applicable, values are expressed in billions of dollars and to four decimal places

Table AIII.
CSM values for debt choices with growth

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$$I = \text{Interest} = \frac{r_D D}{(1 - T_D)} = \frac{0.0662\,(5,216,049,382.72)}{(1 - 0.15)}$$

$$= \$406,238,198.98 \ \text{or about} \ \$0.4062\text{B}.$$

$$g_L = \frac{r_L(1 - T_C)RE}{C + G - \frac{I}{(1-T_C)}} \rightarrow$$

$$g_L = \frac{0.1328\,(1 - 0.3)\,\$578,947,368.42}{\$1,075,187,969.92 + \$218,817,110 - (\$406,238,198.98/(1 - 0.03))}$$

$$= 0.075412081 \ \text{or about} \ 7.541\text{percent}.$$

$$r_{L_g} = r_L - g_L = 0.1328 - 0.075412081 = 0.05738792 \ \text{or about} \ 5.739\text{percent}.$$

(c) A negative g_L value suggests that the debt choice accompanying that value would not be undertaken because a firm would not choose to grow negatively.

(Note: the CSM with growth develops the concept of "g_L" so as to express how a firm's growth in cash flows to equity changes when interest is paid out. Tests, using the CSM equation with growth, suggest that g_L will increase until too much debt causes financial distress problems at which point the computation of g_L breaks down and it becomes negative. While the DVM becomes non-functional when growth rates become too large, the CSM becomes non-functional when g_L becomes negative. If g_L does become negative when too much debt is issued causing $G_L < 0$ to hold, then g_L will reach a point before it becomes negative where V_L will be maximized. Based on the G values in Table AIII, we would expect this maximum V_L to occur before the debt choice of $P = 0.60$ is achieved because this is when G first becomes negative and thus g_L would be expected to become negative. Consequently, one would expect the maximum D to be somewhere from zero debt to \$5.0000B in debt. The solutions to the next question will confirm that this expectation holds.)

Appendix 4. Solutions to question 4

(a) We fill in all empty cells in Table AIV. We see that the maximum G_L is \$2.5356B that occurs in the "0.5" column represented by a debt choice (or P-value) of 0.5. This column also contains the greatest V_L of \$12.9677B and gives the optimal debt-to-firm ratio (D/V_L) of 0.4022.

Below we illustrate the computations in Table AIV for the "0.5" column, which is the column where UGI maximizes its value by issuing \$5.2160B in debt to retire one-half its equity. Using the CSM equation for G_L with growth and inserting given (and previously computed) values, we have:

	P = Debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
Variables	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
V_U (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
$D = P(V_U)$	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
r_D (%)	5.06	5.30	5.60	6.02	6.62	7.34	8.18	9.14	10.28
r_{Lg} (%)	6.790	6.717	6.632	6.399	5.739	23.45	24.41	25.23	25.82
G_L	0.5326	1.0114	1.4110	1.8429	*2.5356*	−2.6564	−2.1150	−1.6176	−1.1985
V_L	10.9647	11.4435	11.8431	12.2750	*12.9677*	7.7757	8.3171	8.8145	9.2336
D/V_L	0.0951	0.1823	0.2643	0.3399	*0.4022*	0.8050	0.8780	0.9468	1.0168

Note: Where applicable, values are expressed in billions of dollars and to four decimal places

Table AIV. CSM values for debt choices with growth

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$$G_L(0.5\ \text{column}) = \left[1 - \frac{\alpha r_D}{r_{L_g}}\right]D - \left[1 - \frac{r_{U_g}}{r_{L_g}}\right]E_U \rightarrow$$

$$G_L(0.5\ \text{column}) = \left[1 - \frac{0.7823529411765(0.0662)}{0.057387919}\right]\$5,216,049,383$$

$$- \left[1 - \frac{0.068538462}{0.057387919}\right]\$10,432,098,765 \rightarrow$$

$$G_L(0.5\ \text{column}) = \$508,640,455 - (-\$2,026,969,490) \rightarrow G_L = \$2,535,609,945 \text{ or about } \$2.5356\text{B}.$$

We can compute the value of the levered firm for the \$5.2160B debt value (which is the 0.5 debt choice) by using the equation:

$$V_L(\text{growth}) = \frac{(1 - T_E)(1 - T_C)[C - I]}{r_{Lg}}$$

where $C = (1 - \text{PBR})(\text{CF}_{BT})$; but it is easier doing:

$$V_L(0.5\ \text{column}) = V_U + G_L(0.5\ \text{column}) = \$10,432,098,765 + \$2,535,609,945 \rightarrow$$

$$V_L = \$12,967,708,710 \text{ or about } \$12.9677\text{B}.$$

Finally, we compute the debt-to-firm value ratio for 0.5 debt choice. Doing this, we get:

$$\frac{D(0.5\ \text{column})}{V_L(0.5\ \text{column})} = \frac{\$5,216,049,383}{\$12,967,708,710} \rightarrow \frac{D}{V_L} = 0.4022.$$

(b) In comparing the optimal debt choice and maximizing values in Table AIV (where UGI has chosen growth) with those in Table AII (where UGI does not use growth), we can begin by noting that the same debt choice ($P = 0.5$) maximizes UGI's value even though the optimal D/V_L has fallen a bit from 0.4412 to 0.4022. Most important, we find that growth has caused G_L to increase from \$1.3331B to \$2.5356B and V_L from \$11.3331B to \$12.9677B. Whereas, growth only increased UGI's unlevered value (V_U) by \$0.4231B (from \$10B to \$10.4321B), growth increased levered firm value (V_L) by \$1.6346B. Thus, it appears that growth with leverage is a good choice. However, the question is: "In practice, would UGI choose $P = 0.5$?" Given the dramatic drop-off in V_L with a debt choice of $P = 0.6$, UGI might choose a P somewhere between 0.4 and 0.5 where V_L with growth would still enhance value more than the levered nongrowth situation.

Appendix 5. Solutions to question 5

(a) In Table AV, we find the G_L values computed previously for the MM, Miller and two CSM equations. In comparing the G_L values, we see that the MM equation gives

Table AV. Comparison of values given by four G_L equations

	P = Debt choice (proportion of unlevered equity retired by debt ranging from 0.1 to 0.9)								
G_L model	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
MM's G_L	0.3158	0.6316	0.9474	1.2632	1.5789	1.8947	2.2105	2.5263	2.8421
Miller's G_L	0.2176	0.4353	0.6529	0.8706	1.0882	1.3059	1.5235	1.7412	1.9588
CSM's G_L (nongrowth)	0.5361	0.9531	1.1804	1.2929	1.3331	1.2829	1.2066	1.1276	1.0400
CSM's G_L (growth)	0.5326	1.0114	1.4110	1.8429	2.5356	−2.6564	−2.1150	−1.6176	−1.1985

Note: All values are expressed in billions of dollars to four decimals

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increasing values for G_L implying that more debt is better. The Miller model also gives increasing G_L values albeit the values are smaller due to the greater personal tax disadvantage of debt compared to equity (because $T_D > T_E$ in our exercise). Thus, at least for our personal tax values, the general conclusion for the Miller equation is like the MM equation: the more debt the better. In analyzing Table AV, we see that CSM equation with nongrowth renders numbers consistent with trade-off theory, which predicts rising G_L values until the optimal debt level is reached and at which point G_L values decline. This rise and decline in G_L occurs because a CSM equation allows the costs of capital for debt and equity to increase (as dictated by an increase in financial risk that concomitantly leads to greater systematic risk). When we use the CSM equation with growth, we get results similar to the CSM without growth with these noticeable differences. First, the G_L values using the CSM with growth tend to be much more positive until they become negative. Second, the negative values for the CSM with growth do not worsen with more debt because there is a break down in computing g_L when G_L becomes negative.

(b) One can notice that the first four debt choices using both CSM equations render greater positive G_L values compared to either MM or Miller. The greater positive values can be attributed not only to the positive tax shield effect but to the fact r_D is less than r_L in the first component of the CSM nongrowth equation (and less than r_{Lg} in the first component of the CSM growth equation for lower debt choices). This can be seen by setting $\alpha = 1$ so as to make the net tax effect zero. Even for this situation, the first components of CSM equations can still generate positive values. Hull (2007) suggests that these positive values can be attributed simply to the way ownership claims are packaged and sold to shield the firm from agency costs.

(c) As a financial manager, you want an equation capturing all of the leverage-related effects. Whereas the MM equation is very simple and thus commonly referred to when discussing the advantage of debt, you might feel more comfortable with an equation (like a CSM equation) that is capable of capturing:

(i) the positive effects that go beyond a tax shield effect; and

(ii) the negative effects of debt.

After the optimal debt level is reached, G_L values using the CSM equation with nongrowth decline because the negativity of its second component begins to dominate; for the CSM equation with growth, negativity occurs for each subsequent debt choice once G_L becomes negative even though the decline does not become increasingly worse (due to the CSM becoming non-functional when g_L becomes negative). For this paper's exercise, relying on the MM and Miller equations causes a firm to issue too much debt. In practice, a firm might make its debt choice based on a desired bond rating. This debt choice is much likely to be consistent with a debt choice recommended by the CSM equations for G_L than by the MM and Miller equations for G_L. For a growth firm using a CSM equation, a manager would have to be careful choosing too much debt due to the steep drop-off in value that occurs. This is consistent with the general belief that growth firms do not like a lot of debt.

Appendix 6. Solutions to question 6

(a) Below we show that UGI would not want to choose a PBR less than 0.30 because it would lower its unlevered firm value. Let us choose PBR = 0.25 to illustrate. To compute V_U, we first compute g_U. For PBR = 0.25, we have:

$$\begin{aligned}RE &= PBR(CF_{BT}) = 0.25(\$1,654,135,338) = \$413,533,835 \text{ and } C \\ &= (1 - PBR)(CF_{BT}) = (1 - 0.25)(\$1,654,135,338) = \$1,240,601,504.\end{aligned}$$

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Thus:

$$g_U = \frac{r_U(1-T_C)RE}{C} = \frac{0.11(1-0.3)\$413,533,835}{\$1,240,601,504}$$

$$= 0.02566666667 \text{ or about } 2.5667\text{percent.}$$

Following the above procedure but using PBR = 0.30, we can get: $g_U = 0.0330$ or 3.30 percent. We now compute V_U values for PBR = 0 (nongrowth), 0.25, and 0.30. We have:

$$V_U(\text{nongrowth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U}$$

$$= \frac{(1-0.05)(1-0.3)(1-0)\$1,654,135,338}{0.11-0.00} = \$10,000,000,000.$$

$$V_U(\text{growth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U}$$

$$= \frac{(1-0.05)(1-0.3)(1-0.25)\$1,654,135,338}{0.11-0.02566666667} = \$9,782,608,696.$$

$$V_U(\text{growth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U}$$

$$= \frac{(1-0.05)(1-0.3)(1-0.30)\$1,654,135,338}{0.11-0.0330} = \$10,000,000,000.$$

Thus, we see that a PBR ratio under 0.30 decreases unleveraged firm value, while a PBR of 0.30 maintains the same unleveraged firm value as the nongrowth situation where PBR = 0. We can use other PBR values less than 0.30 to illustrate why a firm seeking to grow through internal equity cannot choose a PBR under 0.30. For example, PBRs of 0.01, 0.15, and 0.29 give V_U (growth) values of \$9,970,498,474, \$9,697,986,577, and \$9,942,800,789. In conclusion, UGI should not undertake growth with internal equity as an unlevered firm unless it can attain an unleveraged growth rate (g_U) of at least 0.0330, which is the g_U associated with a PBR of 0.30.

(b) As seen earlier in Table VIII, we see that it is possible to identify a PBR that maximizes V_L. For example, a PBR of 0.50 would generate the highest V_U value of \$16.6667B. For this PBR, you would issue no debt. The question a manager must face is whether a growth rate of 7.70 percent is sustainable. For the short term, such a growth rate can be sustained; however, for our perpetuity model (where the growth rate extends for a long period of time), it is likely that 7.70 percent as well as the larger g_U values given in Table VIII are unsustainable by companies. For example, the average growth rate across countries is typically given at a percentage under 4 percent. If that is the case, then most of the PBR choices in the first column of Table VIII are not feasible. UGI has assumed that a growth rate of about 4.15 percent is sustainable given its choice of PBR = 0.35. With this PBR, UGI would maximize firm value with a debt choice of $P = 0.50$, thus retiring one-half of its unlevered firm value (V_U). By issuing this much debt, UGI actually succeeds in having its remaining equity owner's cash flow grow by a larger rate as indicated by its g_L of 7.54 percent.

(c) Given a firm's constraints over how much it can grow and assuming no rigid constraints on debt borrowing, values in Table VIII (given earlier) indicate there is one plowback-payout choice and one debt-equity choice that together maximize firm value.

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Since a greater PBR increases firm value, managers must determine what maximum PBR can be sustained. Once this PBR is chosen, then managers must determine which debt choice combines with this PBR choice to maximize firm value. For example, PBRs of 0.30, 0.35, 0.38, and 0.50 all generate different optimal debt choices. Thus, the choice of PBR determines the debt choice and we can say that the plowback-payout choice determines the debt-equity choice. On the other hand, suppose managers first pick a target debt choice and then go about determining a PBR choice that maximizes V_L. Given this debt choice, a manager could try all feasible PBRs to find out which one generates the maximum V_L.

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